ON MINIMUM INDUCED POWER OF THE HELICOPTER ROTOR

Daniel A. Wachspress
Continuum Dynamics, Inc.
Ewing, New Jersey
dan@continuum-dynamics.com

Todd R. Quackenbush
Continuum Dynamics, Inc.
Ewing, New Jersey
todd@continuum-dynamics.com

Christine L. Solomon
Continuum Dynamics, Inc.
Ewing, New Jersey
christine@continuum-dynamics.com

ABSTRACT

This paper describes research related to minimizing the induced power of a helicopter rotor. The influence of various parameters was studied, including advance ratio, blade twist, blade number, blade cutout, shaft angle, compressibility, stall model, trim condition and higher harmonic cyclic control inputs. Minimum induced power for ideal circulation distributions was also studied. The research was performed by applying state-of-the-art rotorcraft analyses with full-span free-vortex wake models and optimization algorithms. It was observed that the minimum induced power is considerably above the ideal at high advance ratio, roughly a factor of two at μ=0.4 and a factor of three at μ=0.5. Results suggest that even if direct tailoring of the circulation distribution were possible, the minimum induced power would not drop much beyond that achievable with ideally twisted blades and higher harmonic control.

NOMENCLATURE

b	number of blades
c	blade chord
C_{Lmax}	maximum 2D lift coefficient
C_P	power coefficient, $P/\rho\pi R^2(\Omega R)^3$
C_{Pi}	induced power coefficient, $P_i/\rho\pi R^2(\Omega R)^3$
C_{Pp}	parasite power coefficient, $P_p/\rho\pi R^2(\Omega R)^3$
C_Q	torque coefficient, $Q/\rho\pi R^3(\Omega R)^2$
C_{Qi}	induced torque coefficient, $Q_i/\rho\pi R^3(\Omega R)^2$
C_T	thrust coefficient, $T/\rho\pi R^2(\Omega R)^2$
F_M	Figure of Merit, F_M=.7071$C_T^{3/2}/C_Q$
H	H-force or rotor drag force (positive rearward in rotor plane)
M	Mach number
P	rotor power
P_i	rotor induced power
P_o	rotor profile power
P_p	rotor parasite power
P_s	rotor shaft power
Q	rotor shaft torque
Q_i	rotor shaft induced torque
R	rotor radius
U or V	free stream velocity, fps
W	downwash velocity through the rotor disc, fps
X	X-force (positive rearward in wind-axes)
α_S	shaft angle of attack
Γ	circulation
λ	non-dimensional downwash, $\lambda = \mu\tan\alpha_s + \lambda_i$
λ_i	non-dimensional induced downwash, $\lambda_i = W/\Omega R$
μ	advance ratio, $U\cos\alpha_s/\Omega R$ or $V\cos\alpha_s/\Omega R$
Ω	rotor rotation rate, rad/sec
ρ	air density
σ	rotor solidity, $bc/\pi R$
ψ	blade azimuth angle

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INTRODUCTION

Induced power constitutes the majority of total power required by a rotor in hover and is a key contributor to power requirements in forward flight even at significant forward speeds. Reducing induced power in hover is the most effective way to increase the payload and range (through additional fuel carrying capacity) of the helicopter. Reducing induced power in forward flight is an important way to increase range and cruise speed. Thus, there is a strong motivation for understanding how induced power might be minimized in both hover and forward flight.

Classical momentum theory suggests that minimum induced power in hover corresponds to uniform downwash, uniform circulation and triangular loading (increasing from root to tip). This assumes an infinite number of blades, and does not account for the vortical wake trailing from individual blades. To address this, investigations into minimum induced power of rotors and propellers using vortex theory began with work by Betz, Prandtl and Goldstein in the late 1920's. This work is summarized in Refs. 1 and 2 (p.72-92). Betz originally solved the problem for an infinite number of blades and then Prandtl derived a correction to account for tip losses and finite blade number. Goldstein modeled the wake as rigid, helical vortex sheets descending axially downstream. With the advent of high speed computers, numerical vortex wake modeling methods could be used to address the problem. Ref. 2 cites work by Ref. 3 comparing a wide range of methods