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EFFICIENT HIGHER ORDER COMPACT SCHEME FOR DEFORMING UNSTRUCTURED MESHES

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Abstract

Recently, there has been much interest in extending finite volume (FV) based flow solvers to higher order. Previous attempts at achieving higher order accuracy have focused mainly upon: i) finite difference techniques that are restricted to near-uniform structured grids where high order interpolations can be readily developed; and ii) K-order reconstruction techniques which are expensive and prone to round-off problems at high orders and also subject to implementation difficulties near discontinuities and boundaries. Recently, a novel higher order compact (h.o.c.) scheme [1-4] was formulated and implemented upon unstructured grids for application to steady state inviscid flows. The scheme generalizes to arbitrary order accuracy and retains the natural upwinding properties afforded by characteristics-based solvers. The goals of the present paper are i) to extend the formulation to unsteady problems solved upon deforming meshes; ii) to develop an implicit strategy, and; iii) substantially reduce the CPU and storage requirements of previous h.o.c. implementations. Significant advances toward satisfying these objectives have been made and the resulting h.o.c. methodology has been incorporated into an existing 2D Euler analysis. Numerical results generated with the analysis are presented and compared to conventional approaches.

Nomenclature

$[A_j]_{k\ell}$	flux Jacobians associated with edge, j, defined in Eq.(27)
$[\mathbf{B}]_{k\ell}$	third order flux derivative tensor defined in Eq.(28)
e	specific total energy
[J]	Jacobian matrix defined in Eq.(14)
K	polynomial order of accuracy used in the flow state representation
K_k	order of polynomial, T_k
L_1, L_2	baryocentric coordinates of the triangle
$\underline{L}$	vector of baryocentric coordinates, $\underline{L}=\{L_1, L_2\}^T$
$\underline{\hat{n}}$	surface or edge normal vector
N_K	number of polynomials for complete K-accurate flow state representation; in d dimensions, $N_K=(K+d)!\,/\,(K!\,d!)$
[Q]	flux matrix defined in Eq.(2).
$\underline{\overline{Q}}_n$	surface flux vector defined in Eqs.(10)
$\underline{R}$	position vector
S	surface of a FV
t	time
P	static pressure
$\underline{T}$	vector of approximation polynomials
$\underline{u}$	velocity vector with components, {u v w}
$\underline{\overline{U}}$	velocity relative to moving mesh, $\underline{\overline{U}} \equiv \underline{u} - \underline{\dot{X}}$
$\overline{U}_n$	flux velocity, $\overline{U}_n = \underline{\overline{U}} \cdot \underline{\hat{n}}$
V	cell volume
$\underline{W}$	vector of conserved variables
X, Y, Z	Cartesian coordinates
$\underline{\dot{X}}$	mesh velocity vector with components, $\{\dot{X}\ \ \dot{Y}\ \ \dot{Z}\}$
$\underline{Z}_{i,k}$	residual represented in Eq.(20)
γ	ratio of specific heats for air
ϕ_i	test or weighting function
ρ	density
$\underline{\omega}_\alpha$	vector of coefficients used to represent component, α, of the conservation vector
$\underline{\tilde{\omega}}_{i,k}$	vector of coefficients representing the contribution from polynomial, T_k, to the conservation vector, $\underline{W}_i$, of cell, i
$\nabla(\bullet)$	gradient (del) operator
$\underline{(\bullet)}$	(underline) vector quantity
$[\bullet]$	(square brackets) matrix quantity
$(\dot{\ })$	differentiation w.r.t. time
$(\bullet)^T$	transpose operation
$(\bullet)_\alpha$	denotes component, α, of the vector of conservation variables or fluxes
$(\bullet)_n$	quantity resolved along outward normal, $\underline{\hat{n}}$
h.o.c.	higher order compact
l.s.r.	least squares reconstruction

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