A CLOSED-FORM REPRESENTATION OF THREE-DIMENSIONAL STATE-SPACE INFLUENCE COEFFICIENTS

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Abstract

State-space modeling of the rotorcraft flow field is well recognized for its advantage in real-time simulation, preliminary design, eigenvalue analysis, etc. Based on different methodologies, many versions of the state-space model have been developed over the years. From the simplest, crudest approximation of the relationship between on-disk induced flow and thrust perturbations in the 1950s to the most recent studies, which have shown success in representing the induced flow field everywhere above the rotor plane even with mass source terms, the state-space modeling of rotorcraft flow field has become more and more complete and efficient.

In all of the state-space representations, there is a set of influence coefficients between loading distributions and inflow distributions. This matrix is in closed-form as a function of wake skew angle, but it must be inverted numerically to be used in the model. For problems of flight simulation, this implies that the matrix must be inverted at each time step because the wake skew angle is dynamic. Although this inversion is presently done routinely in real-time simulations, it does limit the number of states that can be included. What is needed is a closed-form of the inverse of the matrix to give better facility in numerical applications.

In this work, we utilize an alternative derivation procedure in order to develop directly the $\left[\tilde{\mathbf{L}}\right]^{-1}$ matrix from the Galerkin approach. An analogy in structural dynamics might be that we are aiming at obtaining the stiffness rather than the flexibility matrix. In past work, this inverse has been formed in axial flow and edgewise flow, Ref [1], but never for arbitrary wake skew angles. Here, we develop the closed-form inverse and use it in the dynamic wake model, including comparisons with use of the numerical inverse.

Introduction

State-space modeling is well-known in rotorcraft research and has been continuously developed through the years. This method has the advantages of small computation effort and ease of implementation in stability analysis.

Among many versions of the state-space model, The Peters-He model has the ability to treat on-disk induced flow and pressure (thrust) at any inflow angle from zero to 90 degrees, Ref [2]. The cosine component of this state-space model is expressed as

$$[\mathbf{K}]\{\overset{*}{\alpha}\}+V\left[\tilde{\mathbf{L}}\right]^{-1}\{\alpha\}=\frac{1}{2}\{\tau\} \qquad (1)$$

where $\{\alpha\}$ and $\{\tau\}$ are vectors of velocity and pressure expansion coefficients, respectively.

$$\begin{aligned} u_z &= \sum_{m=0}^{\infty}\sum_{n=m+1,m+3,\cdots}^{\infty} \alpha_n^m \frac{1}{\nu}\bar{P}_n^m(\nu)\cos(m\bar{\psi}) \\ P &= \sum_{m=0}^{\infty}\sum_{n=m+1,m+3,\cdots}^{\infty} \tau_n^{mc}\bar{P}_n^m(\nu)\cos(m\bar{\psi}) \end{aligned} \qquad (2)$$

All components of matrices $[\mathbf{K}]$ and $\left[\tilde{\mathbf{L}}\right]$ in Eq (1) can be expressed in closed-form.

Despite its advantages over preceding models, the He model has a limitation that it can only analyze the flow field on the rotor disk. On the other hand, many practical applications require the state-space model to treat flow in the off-disk area. To eliminate these limitations, Morillo developed another version of the state-space model, Ref [3], which is hierarchical to the He model, but has the ability to treat all three components of the induced flow anywhere above and on the rotor disk. This model is expressed in a similar way (which will be introduced later) also with an inverse of the influence coefficient matrix, $\left[\tilde{\mathbf{L}}\right]$.

However, this inverse matrix in the Morillo model has two problems: 1) it may be ill-conditioned and therefore its inverse cannot be obtained accurately in the numerical process; and 2) the inverse process needs to be done every single step in time-marching if the inflow angle changes over time, as mentioned before. The first

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