Estimation of Velocities and Roll-Up in Aircraft Vortex Wakes

Alan J. Bilanin* and Coleman duP. Donaldson†
Aeronautical Research Associates of Princeton, Inc., Princeton, N.J.

A nonlinear model is developed which determines the swirling and axial velocities in an aircraft vortex wake, given wing lift and drag distributions. The model is shown to reduce to that given by Betz when the axial velocity is the freestream value. The nonlinear interaction of swirling and axial velocities may lead to velocity distributions which are different from those previously calculated. Qualitatively, drag reduces the axial velocity in the vortex and results in an enlarged vortex radius and, therefore, a reduction in swirl velocity. The inviscid model that predicts that significant changes in the structure of the vortex wake, brought about solely by modification of the drag distribution, may require prohibitively large drag penalties. Theoretical results compare favorably with measurements made by Orloff and Grant. A model is developed to estimate the time to roll up a two-dimensional vortex sheet. Results are presented for the cases of linear, parabolic, and elliptic wing loading.

Nomenclature

a = constant, Eq. (21)
A = wing aspect ratio
b = constant, Eq. (21)
$B(t)$ = see Eq. (38)
c = wing chord
c_d = sectional drag coefficient
c_ℓ = sectional lift coefficient
C_L = wing lift coefficient
$g(t)$ = see Eq. (32)
k = constant, Eq. (17)
$\ell(y)$ = sectional lift exerted on the fluid
L = wing lift
P = pressure
q = dynamic pressure
r = radial coordinate
r_t = vortex radius, $r_t = \bar{y}(0)$
$r(t)$ = radial dimension in which the rolled-up vorticity is found
R = characteristic radius of curvature
s = wing semi-span
S = wing planform area
t = time
u,v,w = velocity components in the x,y,z directions, respectively
U_∞ = freestream speed
V = swirl velocity
x,y,z = Cartesian coordinates
$\bar{y}(y)$ = centroid of shed vorticity
$\bar{y}_v(t)$ = horizontal location of the tip vortex during roll-up
α = angle of attack
γ = vortex sheet strength
δ = Eq. (35)
$\Gamma(y)$ = spanwise circulation distribution
Γ' = vortex circulation
Γ_o = wing root circulation
ζ = dummy variable
η = dummy variable
ρ = fluid density
ω = axial component of vorticity

I. Introduction

DAMAGE to aircraft resulting from encounters with other aircraft wakes is a problem which has been documented in both civil and military aviation. Loss of life and equipment has spurred substantial efforts, in recent years, to determine the hazard associated with vortex wakes. It has become apparent that a more detailed description of the aircraft vortex wake than has hitherto been available is needed. The work reported here is an attempt at this description.

To date, many techniques have been tried in hope of alleviating the wake hazard. Included among these were wing load tailoring[1,2] and various drag devices.[3-5] These techniques have had success in that they appear to deintensify the wake. However, while the relationship between wing load distribution and swirling velocity field is understood,[6-10] the role of drag on wake structure has only been treated to resolve the question of the direction of axial velocity along the vortex centerline.[11-13] When large drag devices, such as spoilers, are used (which may also modify wing loading), the effect on wake structure is not apparent. It is the purpose of this investigation to develop simple models from which quantitative results regarding the roll-up of the vortex wake and the downstream wake structure may be obtained. It will be shown that the drag distribution can significantly alter the structure of the vortex irrespective of viscous processes, but that under cruise conditions this effect is modest. While the models developed here consider only the roll-up of tip vortices, the extension to include "interior" or flap vortices is straightforward and is presented in Ref. 14.

Currently, the techniques available to estimate wake structure may be classified such that they fall into one of three categories[‡]: 1) the induced drag-kinetic energy of swirl method suggested by Prandtl[16] in 1927; 2) calculation of the motion of discrete vortex elements or filaments first done by Westwater[17-20] in 1935; or 3) roll-up as prescribed by Betz[6] in 1932 involving conservation of an approximate invariant of the fluid motion.

The first technique involves calculation of the swirl kinetic energy per unit length of wake and equating this to the induced drag of the aircraft. The calculation requires an assumption as to the nature of the swirling velocity distribution with sufficient free parameters such that circulation about each core and the impulse of the vortex system is preserved. In Prandtl's calculation, vortices were assumed to have uniform vorticity cores and for an elliptically loaded wing, core radius was obtained to be 0.155 the semispan of the wing. While calculations of this nature are straightforward, they do not

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Index categories: Aircraft Aerodynamics, Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Consultant. Associate Member AIAA.

†President and Senior Consultant. Associate Fellow AIAA.

‡The similarity solution found by Kaden[15] is omitted here, since it remains valid only for small times.